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# ON SALMON'S AND MacCULLAGH'S METHODS OF GENERATING QUADRIC SURFACES.

By Prof. H. B. Newson, Lawrence, Kas.

The modular and umbilical methods of generating surfaces of the second degree invented by MacCullagh and Salmon, respectively, have been before the mathematical world for fifty years, and have been reproduced in every treatise on these surfaces written in this time, yet their substantial identity has not been generally recognized. This substantial identity I proceed to show as follows:

Following the presentation of the subject given in Frost's Solid Geometry, Chap. XV, MacCullagh's method leads to the equation

$$x^2 + y^2 + z^2 = e_3^2 \left[ (x - \alpha)^2 \sec^2\theta_1 + (y - \beta)^2 \right], \tag{1}$$

where $\theta_1$ is half the angle between the planes of real circular section; and $e_3$, the constant ratio, can easily be shown to be equal to the eccentricity of the principal section of the quadric through the medium and least axes (supposing the surface to be an ellipsoid). The planes of real circular section are perpendicular to the principal plane through the greatest and least axes, and $\tan^2\theta_1 = - e_1^2/e_3'^2$. (For this notation see Annals, Vol. V, p. 3). Let $\theta_2$ denote half the angle between the planes of imaginary circular section perpendicular to the principal section through the greatest and medium axes. Then $\tan^2\theta_2 = - e_2^2/e_3^2$. Whence it easily follows that $\sec^2\theta_1 = - \tan^2\theta_2$, and $\sec^2\theta_2 = - \tan^2\theta_1$. Now replacing in equation (1) $\sec^2\theta_1$ by $-\tan^2\theta_2$, we have

$$x^2 + y^2 + z^2 = - \frac{e_3^2}{\cos^2\theta_2} \left[ (x - \alpha)^2 \sin^2\theta_2 - (y - \beta)^2 \cos^2\theta_2 \right]. \tag{2}$$

Salmon's umbilical method of generating quadrics leads to the equation

$$x^2 + y^2 + z^2 = k \left[ (x - \alpha)^2 \sin^2\theta_1 - (z - \gamma)^2 \cos^2\theta_1 \right], \tag{3}$$

where $\theta_1$ is the same as above; and $k$, the constant ratio, is easily shown to be equal to $- e_3'^2/\cos^2\theta_1$, where $e_3'^2$ denotes the conjugate eccentricity. Equation (3) then must be written

$$x^2 + y^2 + z^2 = - \frac{e_3'^2}{\cos^2\theta_1} \left[ (x - \alpha)^2 \sin^2\theta_1 - (z - \gamma)^2 \cos^2\theta_1 \right]. \tag{4}$$

Replacing $\tan^2\theta_1$ by its equal $- \sec^2\theta_2$ we get

$$x^2 + y^2 + z^2 = e_3'^2 \left[ (x - \alpha)^2 \sec^2\theta_2 + (z - \gamma)^2 \right]. \tag{5}$$

This establishes the identity of the two methods. For $e_3^2$ according to the ordinary conception of the eccentricity of a conic passes through all real values between zero and infinity. When it becomes negative, it is then the square of what I have elsewhere called the conjugate eccentricity and denoted by $e_3'$. Hence, in MacCullagh's modular method, if the square of the modulus be conceived to pass through the complete cycle of real values, and our conception enlarged to include imaginary, as well as real, planes of circular section; then Salmon's method appears as a particular case of MacCullagh's; viz.: when $e_3^2$ is negative. It must be remembered that when $e_3^2$ changes to $e_3'^2$, $\theta_1$ becomes $\theta_2$, and the planes of real circular section revolve through 90°, so that in (5) $y$ and $z$ should be interchanged.

By comparing equation (2) and (4), it may be argued in the same way that MacCullagh's method is only a particular case of Salmon's. Thus they are mutually inclusive, and there is no reason for regarding one more general than the other.

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## SOLUTIONS OF EXERCISES.

### 333

SHOW that

$$\sin\theta > \theta - \frac{\theta^3}{3!} + \frac{1}{45}\left[\frac{\theta^5}{2^2} - \frac{\theta^7}{2^9} + \dots (-)^{m+1}\frac{\theta^{2m+3}}{2^{\frac{1}{2}(m^2+9m+6)}} \pm \dots\right];$$

the general term being the $m$th within the brackets. [*W. H. Echols.*]

SOLUTION.

THE general term as stated in the exercise is wrong; it should be

$$(-1)^{n+1}\frac{\theta^{2n-1}}{(2^2-1)(2^4-1)\dots(2^{2n-2}-1)2^{n-1}};$$

where $n$ is the number of the term in the series.

We have Euler's formula

$$\text{Sin}\,\theta = 2^n \sin 2^{-n}\theta \cos\tfrac{1}{2}\theta\cos\tfrac{1}{4}\theta\dots\cos 2^{-n}\theta.$$

When $n = \infty$ this becomes

$$\text{Sin}\,\theta = \theta\cos\tfrac{1}{2}\theta\cos\tfrac{1}{4}\theta\dots ad.\ inf.$$

$$= \theta(1 - 2\sin^2\tfrac{1}{4}\theta)(1 - 2\sin^2\tfrac{1}{8}\theta)\dots.$$